

# Train Operation and Stop Optimization Model Based on LINGO Solving

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**Abstract:** Train operation arrangement is a key issue in railway operation, and a reasonable arrangement plan can not only meet the needs of passengers, but also reduce operating costs. Based on the passenger flow demand in a certain period, the various costs of train operation are comprehensively considered from the perspective of linear programming, and the optimization model of the train operation scheme is established with the goal of minimizing the cost under the condition of satisfying the transportation demand. The optimization results show that the optimization scheme given by the model is more flexible and practical in the selection of departure and stopping stations than the current operation scheme, and greatly reduces the operation cost, and the given decision-making scheme meets the actual dynamic passenger flow demand.

**Keywords:** Trains operation; Linear programming; Dynamic passenger flow.

## 1. Introduction

Train operation mainly considers the cost of running and the number of passengers between stations, generally speaking, the cost of running is relatively fixed, the number of passengers between the two stations will show a strong timeliness change, part of the period of crowding, part of the period of sparse flow of people, how to dynamically adjust the train operation scheme according to the flow of people, to achieve both to meet the needs of passengers, but also to make the train operation cost to the minimum has been a problem for railway operating companies and scholars.

Zhou Jiaying et al.[1] considered minimizing the operating cost of the enterprise as much as possible and maximizing the passenger service level as much as possible to solve the problem of train operation scheme in the large and small intersection mode. Li Bing et al.[2] proposed three modes of train operation at the loading station, and with the goal of minimizing the hourly consumption, considering the limitations of the only operation mode, the only path selection, the adaptation ability of the technical station and the ability to pass through the section, the average daily traffic consumption of the loading station during the statistical period, the optimization model of the combined operation scheme of the loading station based on the average daily traffic consumption was constructed. Tian Peining et al.[3] aimed at maximizing the number of cross-line direct passengers, the minimum total travel time of passengers, and the minimum number of vehicle kilometers, and constructing a cross-line train operation scheme planning model between the ring line and the radius line, considering the constraints of departure interval, full load rate, and number of trains. Dong Xiao et al.[4] designed the transportation service network, and established an optimization model for the operation scheme of non-coal cargo trains on the main coal transportation line with the goal of maximizing the revenue of the railway department of the National Energy Group. Wu Tianqi et al.[5] analyzed the advantages of urban trains in Fuzhou metropolitan area by combing the mechanism of coordinated development of urban railways and metropolitan areas at home and abroad, drawing on the experience of urban trains at home and abroad, and combined with the development planning, spatial and industrial layout of Fuzhou metropolitan area and the actual needs of urban trains, they constructed an existing railway operation organization and technical system with "opening up, dense up, and quality lifting" as the core. Xiao Jie et al.[6] considered the two forms of traffic flow organization of single trains and group trains, and established an optimization model for train operation scheme with the number of shunting lines and the adaptation capacity of technical stations as the limiting conditions, and the minimum total cost of traffic flow organization as the optimization goal.

In this paper, we dynamically consider the changes in passenger flow, divide the passenger flow into demand segments according to time breaks, and establish an optimization model for station stops in this period based on the passenger flow demand in a certain period and the actual train operation, so as to obtain the operation plan of the corresponding period.

## 2. Model Building

### 2.1 Description of the symbols

Table 2.1 Symbols and descriptions used in the model

Symbol	Description of the symbols
$U$	Represents the capacity of the vehicle, that is, the maximum number of people that the train can carry
$S$	Represents the set of all train stations, and the subscript of station elements is represented by $i, j$
$T$	Represents the set of all vehicles, and the subscript of the train element is represented by $k$
$P_k$	Represents the initial number of people on vehicle $k$
$s_i$	Represents whether the depot $i$ can be used as the departure depot
$m_i$	Represents whether station $i$ can be used as a terminal site
$C_i$	Represents the cost of stopping at station $i$
$O_k$	Represents the cost of driving vehicle $k$
$R_k$	Represents the transportation cost per unit distance per unit of vehicle $k$
$L_i$	Indicates the maximum number of trains that can be departed at station $i$
$U_i$	Represents the maximum number of parking spaces that can be accommodated at station $i$
$OD_{ij}$	Represents the number of passengers from station $i$ to station $j$
$b_{ij}$	Represents whether there is a transportation requirement from station $i$ to station $j$ ( $b_{ij} = 0$ No, $b_{ij} = 1$ Yes)
$d_{ij}$	Represents the transportation distance from station $i$ to station $j$
$w_{ki}$	Represents whether vehicle $k$ can use the $i$ -th point as the departure point
$r_{ki}$	Represents whether vehicle $k$ can use the $i$ -th point as the final point
$z_k$	Represents whether or not to arrange vehicle $k$ to serve passengers
$x_{ki}$	Indicates whether the vehicle $k$ stops at the $i$ -th point
$y_{kij}$	Indicates whether vehicle $k$ serves the passenger demand from station $i$ to station $j$
$q_{kij}$	Represents the number of passengers from vehicle $k$ assignment service station $i$ to station $j$
$s_{ki}$	Indicates the number of passengers on the vehicle when vehicle $k$ departs from point $i$
$u_{ki}$	Indicates the number of people in the vehicle when the vehicle $k$ reaches the $i$ -th point

### 2.2 Model Building

In this paper, the following optimization model is established

$$\min \sum_{k \in T} z_k \cdot O_k + \sum_{k \in T} \sum_{i \in S} C_i \cdot (x_{ki} - w_{ki}) + \sum_{k \in T} \sum_{i \in S} \sum_{j \in S, i < j} R_k \cdot d_{ij} (q_{kij} + P_k) \quad (1)$$

s.t.

$$\sum_{i \in S} x_{ki} \geq 2z_k, \forall k \in T \quad (2)$$

$$w_{ki} \leq x_{ki}, \forall k \in T, i \in S \quad (3)$$

$$r_{ki} \leq x_{ki}, \forall k \in T, i \in S \quad (4)$$

$$w_{ki} \leq s_i, \forall k \in T, i \in S \quad (5)$$

$$\sum_{i \in S} w_{ki} = z_k, \forall k \in T \quad (6)$$

$$r_{ki} \leq m_i, \forall k \in T, i \in S \quad (7)$$

$$\sum_{i \in S} r_{ki} = z_k, \forall k \in T \quad (8)$$

$$x_{ki} \leq z_k, \forall k \in T, i \in S \quad (9)$$

$$x_{ki} \leq 1 - w_{kj}, \forall k \in T; i, j \in S, i < j \quad (10)$$

$$x_{kj} \leq 1 - r_{ki}, \forall k \in T; i, j \in S, i < j \quad (11)$$

$$y_{kij} \leq x_{ki}, y_{kij} \leq x_{kj}, \forall k \in T; i, j \in S, i < j \quad (12)$$

$$y_{kij} = 0, \forall k \in T; i, j \in S, i \geq j \quad (13)$$

$$\sum_{k \in T} y_{kij} \geq b_{ij}, \forall i, j \in S, i < j \quad (14)$$

$$\sum_{k \in T} q_{kij} = OD_{ij}, \forall i, j \in S, i < j \quad (15)$$

$$q_{kij} \leq Uy_{kij}, \forall k \in T; i, j \in S, i < j \quad (16)$$

$$s_{km} = u_{km} + \sum_{j \in S, m < j} q_{kmj} - \sum_{i \in S, i < m} q_{kim}, \forall k \in T; \forall m \in S \quad (17)$$

$$u_{ki} \geq P_k w_{ki} - M(1 - w_{ki}), \forall k \in T, \forall i \in S \quad (18)$$

$$u_{ki} \leq P_k w_{ki} + M(1 - w_{ki}), \forall k \in T, \forall i \in S \quad (19)$$

$$s_{km} = u_{k, m+1}, \forall k \in T; \forall m \in S, m \neq |S| \quad (20)$$

$$s_{km} \leq U, \forall m \in S, \forall k \in T \quad (21)$$

$$x_{km} \leq \sum_{j \in S, m < j} y_{kmj} + \sum_{i \in S, i < m} y_{kim}, \forall m \in S, \forall k \in T \quad (22)$$

$$\sum_{k \in T} (w_{ki} + x_{ki} - r_{ki}) \leq L_i, \forall i \in S \quad (23)$$

$$\sum_{k \in T} (x_{ki} - w_{ki}) \leq U_i, \forall i \in S \quad (24)$$

$$x_{ki}, w_{ki}, r_{ki}, y_{kij}, z_k \in \{0, 1\}, \forall i, j \in S, k \in T \quad (25)$$

$$s_{ki}, u_{ki}, q_{kij} \in N, \forall i, j \in S, k \in T \quad (26)$$

In the optimization model, (1) indicates that the optimization goal is to minimize the comprehensive cost, which includes the cost of train operation, the cost of vehicle stopping, and the cost of transportation per unit distance. (2) indicates that the train that arranges to pick up and drop off needs to stop at least 2 stations; (3) indicates that the train can only depart from the station where it stops, or the station where it departs must be the station where it stops; (4) indicates that the train can only select the final station from the stopping station, or the final station must be the

stopping station; (5) indicates that the departure can only be selected at the station that can provide the departure conditions; (6) indicates that each train can only choose one station from all the selectable stations to depart; (7) indicates that the final stop can only be selected at the station that can provide the final arrival condition; (8) indicates that each train can only select one station as the final station among all the selectable stations; (9) indicates that trains with stops can only be scheduled for this batch decision; (10) indicates that the station that arrives later on the driving route is the starting station of this batch, then the station in front cannot be stopped; (11) indicates that the station that arrives first on the driving route is the final station of this batch, then the later station cannot be stopped; (12) indicates that the parked vehicle needs to transport the passenger flow from the stopping station to other stations; (13) indicates that at least one train stops at least one train at two stations with passenger flow demand for transportation; (14) indicates that the transportation demand should be fully satisfied; (15) indicates that the transportation of the train does not exceed the upper limit of the transportation capacity of the train; (16) indicates that the number of people on the bus when each car departs from a certain point is equal to the number of people on the bus when it arrives at the station, plus the number of people on the bus minus the number of people who get off the bus; (17) and (18) indicate that the number of people when the vehicle passes through the departure station is determined by whether it stops or not; (19) to (22) indicate the restrictions on the number of stops and departures related to each station; (23) indicates that the decision variable is an integer or a 0-1 variable constraint.

### 3. Instance Validation Solution

#### 3.1 Study Data Settings

The example data used in this chapter are from the passenger flow statistics of Lanzhou Metro Line 1[21], there are 10 buses available for OD transportation that cannot be arranged for passenger flow during the time period, and 9 stations can provide departure and parking, and the relevant parameter data of each station are shown in Table 3.1.1.

Table 3.1.1 Station related parameter data

	Whether it can be used as a departure station $s_i$	Stop Cost $C_i$ (RMB/time)	The maximum number of departures is $L_i$	Reception maximum number $U_i$	Terminal Site $m_i$
West Station Cross	1	800	150	90	0
Seven Mile River	0	800	150	90	0
Little West Lake	0	800	150	90	0
Palace of Culture	0	800	150	90	0
Xiguan	0	800	150	90	0
Provincial government	0	800	150	90	0
Dongfanghong Square	0	800	150	90	0
Lanzhou University	0	800	150	90	0
Five-mile shop	0	800	150	90	0
Provincial Meteorological Bureau	0	800	150	90	1

The OD traffic between the upbound stations in a study period is shown in Table 3.1.2.

Table 3.1.2 OD Passenger Flow Between Stations (Unit: Person)

Stop	West Statio n	Seve n Mile	Littl e Wes	Palace of Cultur	Xigua n	Provincial governme nt	Dongfangho ng Square	Lanzhou Universit y	Five-mil e shop	Provincial Meteorologic al Bureau
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	Cross	River	t Lake	e						
West Station		304	200	787	61	413	201	151	298	254
Cross										
Seven Mile			215	522	302	154	702	292	282	414
River										
Little West				403	42	58	452	282	212	216
Lake										
Palace of					62	76	52	122	246	356
Culture										
Xiguan						80	76	412	212	147
Provincial							453	523	352	323
government										
Dongfangho								252	363	247
ng Square										
Lanzhou									216	126
University										
Five-mile										414
shop										

The distance between stations is shown in Table 3.1.3.

	West Stati on Cros s	Sev en Mil e Riv er	Litt le We st Lak e	Palac e of Cultu re	Xigu an	Provinci al governm ent	Dongfangh ong Square	Lanzho u Univers ity	Five-m ile shop	Provincial Meteorolog ical Bureau
West Station Cross		1.6	3.9	5.2	6.8	8.6	9.8	11.2	12.9	14.7
Seven Mile River			2.3	3.6	5.2	7	8.2	9.6	11.3	13.1
Little West Lake				1.3	2.9	4.7	5.9	7.3	9	10.8
Palace of Culture					1.6	3.4	4.6	2	3.7	5.5
Xiguan						1.8	3	4.4	6.1	7.9
Provincial governmen t							1.2	2.6	4.3	6.1
Dongfangh ong Square								1.4	3.1	4.9
Lanzhou University									1.7	3.5
Five-mile shop										1.8
Provincial Meteorolo gical Bureau										

Table 3.1.3 Distance between stations (in km)

The information of the vehicles to be selected is shown in Table 3.1.4

	Initial number of people on the bus Pk (person)	Setup cost Ok (RMB/time)	Transportation cost per unit distance per unit demand Rk (RMB/km)
Train 1	0	700	3
Train 2	0	700	3
Train 3	0	700	3
Train 4	0	800	3
Train 5	0	900	3
Train 6	0	800	3
Train 7	0	700	3
Train 8	0	600	3
Train 9	0	500	3
Train 10	0	600	3

Table 3.1.4 Vehicle Information

### 3.2 Interpretation of the Results

Running the LINGO solver, the arrangement result of solving the train selection is shown in Table 3.2.1.

Table 3.2.1 Model decision-making arrangement scheme (1 indicates arrangement, 0 indicates no arrangement)

Train	1	2	3	4	5	6	7	8	9	10
Arrangement	1	0	0	1	0	1	0	1	1	0

The results show that choosing 5 out of 10 vehicles to serve this batch of OD passenger flow can achieve the lowest overall cost 235654. According to the solution results, the stop scheme of each vehicle is shown in Table 3.2.2.

Table 3.2.2 Decision-making vehicle stop scheme (1 indicates stop)

The number of people on board when each vehicle arrives at each station is shown in Table 3.2.3

	West Station Cross	Seven Mile River	Little West Lake	Palace of Culture	Xiguan	Provincial government	Dongfanghong Square	Lanzhou University	Five-mile shop	Provincial Meteorological Bureau
Train 1	1		1		1	1	1		1	
Train 4	1	1	1	1		1				1
Train 6	1	1					1		1	
Train 8	1		1	1		1	1	1	1	
Train 9	1	1		1	1		1	1	1	1

Table 3.2.3 Number of people on board each vehicle when it leaves each station

	West Station Cross	Seven Mile River	Little West Lake	Palace of Culture	Xiguan	Provincial government	Dongfanghong Square	Lanzhou University	Five-mile shop	Provincial Meteorological Bureau

	r		Lak		e					
Train 1	707	707	968	968	945	1500	765	765		
Train 4	194	1499	1500	1331	1331	1500	1500	1500	1500	
Train 6	47	1030	1030	1030	1030	1030	691	691		
Train 8	512	512	1500	1354	1354	1500	1500	386		
Train 9	1209	1500	1500	1017	1500	1500	1500	922	997	

### 3.3 Comparative Analysis

According to the survey, the current operation scheme is that all the train stations are stopped, and the cost required under the assumed data conditions is 289854, and the number of trains running at this time to each station is shown in Table 3.3.1.

Table 3.3.1 Number of people on board leaving each station under the stop-at-all scheme

	West Station Cross	Seven Mile River	Little West Lake	Palace of Culture	Xiguan	Provincial government	Dongfanghong Square	Lanzhou University	Five-mile shop	Provincial Meteorological Bureau
Train 1	9	354	752	414	363	256	253	248	9	
Train 2	9	16	21	90	91	24	21	222	9	
Train 3	15	22	501	616	988	1500	1497	224	9	
Train 4	9	16	69	72	143	24	616	369	9	
Train 5	730	1500	1500	366	25	24	21	16	204	
Train 6	116	123	128	24	25	24	188	183	176	
Train 7	291	1289	1500	1500	1500	1499	762	544	215	
Train 8	199	406	21	24	25	679	676	706	357	
Train 9	329	740	745	1094	1500	1500	1500	1500	1500	
Train 10	962	782	1261	1500	1500	1500	422	252	9	

Comparing Table 3.2.3, it can be seen that under the model optimization decision-making arrangement scheme, the arranged vehicles are fully utilized, and the cost is reduced by 18.67% compared with the scheme of stopping at all stations, and if the all-day operation period is considered, the operating cost will be greatly reduced.

## 4. Conclusion

The optimization model established in this paper solves the optimal train arrangement scheme in a certain time period, which reflects the timeliness of passenger demand and the flexibility of station departure, the extra train can be used as the alternative train for the next stage of operation decision, and the train that has completed the transportation can also participate in the decision-making of the later stage on a rolling basis, and the optimization model used fully considers the cost of each item and the constraints of the actual stop, which can not only accurately reflect the actual situation of the operation, but also give an effective decision-making reference for the solution results.

The method used in this paper can be extended to the operation of various trains and even city buses, etc., and the robustness of the model in the application scenarios and response to different data scenarios is relatively good, and the conclusions obtained can provide an effective reference for the actual operation, improve the operation efficiency and save the operation cost.

## References

- [1] ZHOU Jiaying; ZHOU Xuanyu; LI Xiaodong. Research on optimization model of large and small intersection scheme based on genetic algorithm[J].Science and Technology Innovation and Application,2023,13(31):63-66.)
- [2] LI Bing; JIANG Shangtao; Xuan Hua. Optimization of multi-mode combined train operation scheme of loading station based on fluctuation of train consumption[J].Journal of Beijing Jiaotong University,1-12.
- [3] TIAN Peining; TONG Ruiyong; WANG Haipeng; MAO Baohua; ZHANG Haoxiang; LU Xia. Optimization of cross-line train operation scheme of urban rail transit cross-ring line[J].Journal of Jilin University(Engineering Science),1-10.
- [4] DONG Xiao; CHAO Zhen; HUANG Sixiang; ZHAO Yangzi; WANG Yu. Research on optimization of non-coal freight train operation scheme of coal transportation trunk line[J].Railway Freight,2023,41(09):40-47.)
- [5] WU Tianqi; YANG Xiao; LIU Xiaoxi; Bao Jingjing; WANG Rui. Research on the strategic path of urban train operation in Fuzhou metropolitan area[J].Strait Science,2023,(09):114-120+126.)
- [6] XIAO Jie; WANG Long; YANG Wenmao; CAO Linwei. Research on optimization scheme of express freight train operation based on ant colony algorithm[J].Integrated Transportation,2023,45(09):136-139+147.)
- [7] YUAN Bo; LI Haiying; LIAO Zhengwen. Coordination and optimization of train timetable and maintenance skylight on high-speed railway[J].Journal of Railway Science and Engineering,1-11.
- [8] XU Dejie; PAN Xing; GONG Liang; Hu Chenhao; WANG Xuexin. Multi-intersection passenger flow allocation method of urban rail transit based on train operation scheme[J].Transportation Systems Engineering and Information Technology,2023,23(05):238-246+289.)
- [9] DU Jianfei; WANG Tianbi; Hu Xinyue; ZHANG Yongxiang; ZHONG Qingwei; ZHAO Tianyin; PENG Qiyuan. Research on optimization of EMU utilization under dynamic demand of train operation[J].Comprehensive Transportation,1-8
- [10] CHANG Xiujuan; Anfei; Guodong plum; Mo Yanxiang; CHEN Yuyan; SUN Yanying; ZHAO Li. Optimization of train operation scheme of Shijiazhuang Metro Line 1[J].Transportation Energy Conservation and Environmental Protection,2023,19(05):270-274.)
- [11] XU Guangming; Lu Chunyu; Zhong Linhuan; DENG Lianbo. Integrated optimization method of urban rail flow control scheme and train running frequency and fare[J].Transportation Systems Engineering and Information,2023,23(05):268-278.)
- [12] FANG Bo; WEI Yuguang; MA Bowen. Collaborative optimization of single group and grouped train marshalling plan of technical station[J].Journal of the China Railway Society,2023,45(07):10-19.)
- [13] LI Xiaotao; Luo Changquan; WANG Rui. Research on optimization of intercity railway train operation scheme under large and small group mode[J].Railway Transport and Economy,2023,45(07):8-13.)



- [14] LIU Mengxuan. Research on the strengthening scheme of passing capacity of Handan-Huang Railway based on running tracking train[D].Shijiazhuang Tiedao University,2023.
- [15] YIN Chuanzhong; LI Yueshan; Tao Xuezhong; LIU Mi. Scheme of railway container express train operation considering carbon emission cost[J].Transportation Information and Security,2023,41(03):128-137.)
- [16] CHEN Lei; Duan Xiaoyu; Bai Yun. Urban Rapid Rail Transit,2023,36(03):59-64.)
- [17] XU Ance. Optimization of the operation scheme of high-speed rail corridor express freight train based on departure time[J].Journal of Lanzhou Jiaotong University,2023,42(03):22-31.)
- [18] WANG Chen; Jia Feifan; Liu. Research on the impact of train operation structure adjustment on railway traffic volume of ordinary speed line[J].Railway Economic Research,2023,(03):23-27.)
- [19] ZHANG Chenggong. Research on optimization scheme of Danda intercity passenger dedicated line train[D].Dalian Maritime University,2022.
- [20] QI Jianguo. Research on collaborative optimization of railway train timetable and stop scheme[D].Beijing Jiaotong University,2019.
- [21] Lanzhou Rail Transit Co., Ltd. Operating timetable [2023-10-15].  
<https://www.lzgdjt.com/lzgd/serve.jsp#timetable>