

The Determination of Effective Modes of Wind-Bridge Coupling System Based on Stabilization Clustering Method

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Abstract. Modal identification is the core content of the full-bridge aeroelastic model wind tunnel test. Modal consistency is the most direct and effective basis to ensure that the model and the real bridge meet the similarity criteria. Accurate, simple and fast identification of modal parameters has very important practical significance for the evaluation and correction of aeroelastic models. In this paper, the eigensystem realization algorithm (ERA) and stochastic subspace identification (SSI) is improved, and the automatic identification theory of effective modes of bridge aeroelastic model based on stable clustering method is proposed. The stable diagram eliminates the unstable modes by comparing the results of different system orders, and the pedigree clustering further polarizes the stable modes to obtain multi-order and high-precision system modes. Numerical simulation examples and wind tunnel engineering examples show that the method has high accuracy and good adaptability, and can solve the problem of unstable damping identification results to a certain extent. In the wind tunnel test, the stable clustering method is one of the effective methods to identify the modal of the bridge aeroelastic model excited by the wind environment.

Keywords: Stabilization diagram; hierarchical clustering; ERA; SSI; effective modes.

1. Introduction

The relationship between system, excitation and response is the core of modal analysis. Finite element dynamic analysis is a dynamic forward problem of establishing a structural model system and solving the output structural response under the input load excitation. Modal analysis is a dynamic inverse problem that collects the excitation load and response of the structural system to deduce the parameters of the structural system. Modal analysis has four steps: data acquisition, data processing, modal identification and model validation. Modal identification is a crucial step. The traditional method is to identify the mode according to the calculation of the frequency response function by using the synchronously observable excitation and response. Due to the uncontrollable excitation and the low identification efficiency, in the wind tunnel model test, The system mode identification method based on input and output is difficult to realize, Therefore, seeking an identification method only based on the output is the focus of many scholars' research^[1]. Of course, the system input is assumed to be white noise or stationary ambient excitation.

Modal analysis is of great significance in the field of bridge wind engineering. Specific performance includes: similarity correction of aeroelastic model and real beam, identification of flutter derivative of bridge section^[2] and dynamic analysis of bridge under wind load. At present, the commonly used modal identification method in wind tunnel test is to obtain the free response signal of aeroelastic model through artificial initial excitation. Then the modal parameters are identified by Fourier transform or unit impulse response function. There are many defects in this method: 1) there are missing orders in the identification of high-order modes; 2) Due to the limited number of measuring equipment, the identification of model vibration mode needs to be completed based on multiple tests of reference points; 3) The instability of damping still exists, and the results of multiple measurements of damping are very discrete. Therefore, this paper combines the inherent wind load excitation source of the wind tunnel laboratory and The non-contact full-field strain and three-dimensional displacement measuring instrument has the advantage of full-field three-dimensional synchronous observation, and through the improvement of ERA and SSI algorithms, The method of combining stability diagram with hierarchical clustering for automatic identification of modal parameters of aeroelastic model of bridge in wind tunnel test is proposed the

stable clustering method), not only improves the efficiency and accuracy of the test, but also solves the problem that the damping identification result is unstable to a certain extent.

2. An Improved Algorithm for Modal Identification

2.1 ERA Algorithm Improvement

The discrete state space equation of n degrees of freedom of the structural vibration system is (L is the number of excitation points, N is the number of measurement points):

$$\begin{cases} \dot{\mathbf{y}}_{2n}(t) = \mathbf{A}_{2n \times 2n} \mathbf{y}_{2n}(t) + \mathbf{B}_{2n \times L} \mathbf{f}_L(t) \\ \mathbf{z}_N(t) = \mathbf{G}_{N \times 2n} \mathbf{y}_{2n}(t) \end{cases} \quad (1)$$

Construct Hankel matrix according to the response data matrix $\mathbf{h}(k)$ of the system:

$$\mathbf{H}(k-1) = \begin{bmatrix} \mathbf{h}(k) & \mathbf{h}(k+1) & \cdots & \mathbf{h}(k+\beta-1) \\ \mathbf{h}(k+1) & \mathbf{h}(k+2) & \cdots & \mathbf{h}(k+\beta) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{h}(k+\alpha-1) & \mathbf{h}(k+\alpha) & \cdots & \mathbf{h}(k+\alpha+\beta-2) \end{bmatrix}_{\alpha N \times \beta L} \quad (2)$$

Set the system to be of order $2n$, decompose the singular values of $\mathbf{H}(0)$ according to Formula $\mathbf{H}(0) = \mathbf{U}_{\alpha N \times 2n} \boldsymbol{\Sigma}_{2n \times 2n} \mathbf{V}_{\beta L \times 2n}^T$. To improve the accuracy, the first $2n$ singular values are filtered and the minimum realization of the system is constructed as follows:

$$\mathbf{A} = \boldsymbol{\Sigma}^{-\frac{1}{2}} \mathbf{U}^T \mathbf{H}(1) \mathbf{V} \boldsymbol{\Sigma}^{-\frac{1}{2}} \quad \mathbf{B} = \boldsymbol{\Sigma}^{\frac{1}{2}} \mathbf{V}^T \mathbf{E}_L \quad \mathbf{G} = \mathbf{E}_N^T \mathbf{U} \boldsymbol{\Sigma}^{\frac{1}{2}} \quad (3)$$

An eigenvalue matrix $\boldsymbol{\Lambda}$ and an eigenvector matrix $\boldsymbol{\Phi}$ can be obtain by decomposing a system matrix \mathbf{A} by an eigenvalue, a sampling time interval $\Delta\tau$ and an observation matrix \mathbf{G} are added. The i -th complex frequency λ_i and complex mode shape ϕ_i of the system are:

$$\lambda_i = \ln(\Delta_i) / \Delta\tau \quad \phi_i = \mathbf{G} \boldsymbol{\Phi}_i \quad (4)$$

Tests show $\pm k$ that further averaging of the system matrix $\mathbf{H}(0)$ obtained by matrix time shifting can reduce noise pollution and improve accuracy. The system matrix $\bar{\mathbf{A}}$ is expressed as follows:

$$\bar{\mathbf{A}} = \frac{1}{2} [\boldsymbol{\Sigma}^{-\frac{1}{2}} \mathbf{U}^T \mathbf{H}(k) \mathbf{V} \boldsymbol{\Sigma}^{-\frac{1}{2}} + \boldsymbol{\Sigma}^{-\frac{1}{2}} \mathbf{U}^T \mathbf{H}(-k) \mathbf{V} \boldsymbol{\Sigma}^{-\frac{1}{2}}] \quad (5)$$

2.2 SSI Algorithm Improvement

The stochastic time discrete state space model of an n -dof structural vibration system is composed of the dynamic differential equation and the observation equation:

$$\begin{cases} \mathbf{y}_{k+1} = \mathbf{A} \mathbf{y}_k + \mathbf{w}_k \\ \mathbf{z}_k = \mathbf{G} \mathbf{y}_k + \mathbf{v}_k \end{cases} \quad (6)$$

Where, \mathbf{A} is a system matrix also called a state space matrix, \mathbf{G} is an observation matrix also called an output matrix, \mathbf{w}_k and \mathbf{v}_k are noise of input and output (including white noise excitation term).

Constructing an Hankle block matrix according to the data \mathbf{z}_k of the N measuring points of the system response:

$$\mathbf{H}_0 = \frac{1}{\sqrt{\beta}} \begin{bmatrix} \mathbf{z}_0 & \mathbf{z}_1 & \cdots & \mathbf{z}_{\beta-1} \\ \mathbf{z}_1 & \mathbf{z}_2 & \cdots & \mathbf{z}_{\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{z}_{\alpha-1} & \mathbf{z}_{\alpha} & \cdots & \mathbf{z}_{\alpha+\beta-2} \\ \mathbf{z}_{\alpha} & \mathbf{z}_{\alpha+1} & \cdots & \mathbf{z}_{\alpha+\beta-1} \\ \mathbf{z}_{\alpha+1} & \mathbf{z}_{\alpha+2} & \cdots & \mathbf{z}_{\alpha+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{z}_{2\alpha-1} & \mathbf{z}_{2\alpha} & \cdots & \mathbf{z}_{2\alpha+\beta-2} \end{bmatrix}_{2\alpha N \times \beta} = \begin{bmatrix} \mathbf{H}_p \\ \mathbf{H}_f \end{bmatrix} \quad (7)$$

Take the QR decomposition of $H_0 = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = RQ^T$, construct the projection matrix $O_\alpha = R_{21}Q_1$ of the row space, and take the singular value decomposition of O_α :

$$O_\alpha = U\Sigma V^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 = 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 \Sigma_1 V_1^T \quad (8)$$

Filtering is also performed on the singular values. The extended observable matrix is $T_\alpha = U_1 \Sigma_1^{1/2}$, and the sum of two sub-matrices is T_1 and T_2 constructed by subtracting rows, then the system matrix $A = (T_1)^{-1} T_2$, the observation matrix G is the first N rows of T_α . The frequency and vibration mode of the system are solved according to the formula (4) after the eigenvalue decomposition of the system matrix.

$$T_1 = \begin{bmatrix} G \\ GA \\ \vdots \\ GA^{2n-2} \end{bmatrix} \quad T_2 = \begin{bmatrix} GA \\ GA^2 \\ \vdots \\ GA^{2n-1} \end{bmatrix} \quad (9)$$

Tests show that weighting the projection matrix O_α can significantly improve the anti-noise performance of the SSI algorithm, \bar{O}_α expressed as follows:

$$\bar{O}_\alpha = (H_f H_f^T)^{-1/2} O_\alpha (H_p H_p^T)^{-1/2} \quad (10)$$

3. Theory of Stable Clustering Method

Stability Chart^[3] Theoretical system: the comparison between the modes of adjacent order systems is realized by adjusting the system order $2n$. Unstable modes (including frequency instability, damping instability, and frequency instability) are defined when either the frequency or the damping exceeds a predetermined threshold Damping is unstable), and the rest are considered as stable modes. In order to improve the accuracy, the modal confidence factor (MAC) and the phase collinear factor (MPC) can be added according to the similarity of the vibration mode. Set the standard.

Pedigree clustering method^[4] Theoretical system: objective objects are grouped into different categories according to their closeness and similarity. Objects in different classes have obvious dissimilarity, and objects in the same class have high similarity. In the identification of system modes, the hierarchical clustering method is introduced to complete the classification of modes (poles) and improve the accuracy of modes (clustering after averaging Class center), the similarity of two modes is weighed by their difference (see Equation 11 for details), and the difference is inversely proportional to the similarity. x can be any combination of modal confidence factor, modal energy, frequency or damping.

$$D_{ij} = \sum \frac{W_x}{dx} \frac{|x_i - x_j|}{\max(x_i, x_j)} \quad (11)$$

Where: W_x — weighting coefficient $\sum W_x = 1$, dx — threshold value. In this paper, the frequency and damping are taken x , the frequency threshold is taken as 0.1 Hz, the damping threshold is taken as 0.2%, and the weighting coefficients are taken as 0.5.

Specific steps of the pedigree clustering method:

1. determining a similarity value between modes according to a formula (11), and constructing a symmetric similarity matrix with a diagonal element of 1;
2. for a fuzzy similarity tree according to that similarity matrix;
 - (1) finding the sum of the two mode subclasses with the maximum similarity M_i and M_j ;
 - (2) connecting M_i and M_j modes as a new subclass $M_{i \text{ new}}$, M_i and M_j eliminating the similarity values of and from the similarity matrix;
 - (3) If all subclasses are merged into one class, stop, otherwise (1) is executed.
3. determine that number of the cluster and the cluster centers to complete the clustering.

The stability diagram can effectively eliminate false modes and unstable modes, The stable modes identified by the stability diagram are clustered and averaged by the pedigree clustering method, The stable clustering method can determine the true mode and improve the accuracy.

4. Numerical Simulation Examples

In order to verify the reliability of the stable clustering method, a simply-supported beam model with a rectangular section is set as a simulation example, and the model is shown in Figure 1. Its parameters are listed in Table 1.

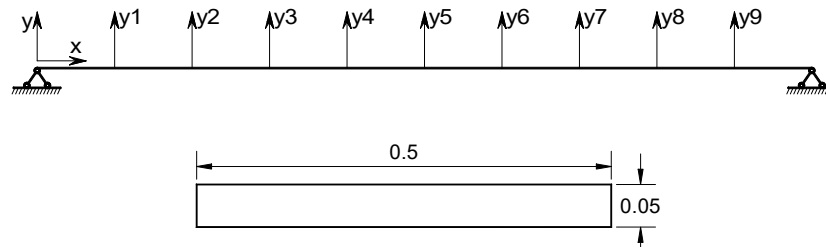


Fig. 1 Simulation model/m

Table 1. Parameter setting of simulation model

Parameter	Numerical value	Parameter	Numerical value
Beam length	5m	Beam width	0.5m
Beam high	0.05m	Cross-sectional area	$2.5 \times 10^{-3} \text{ m}^2$
Bending moment of inertia	$5.21 \times 10^{-9} \text{ m}^4$	Modulus of elasticity of material	$2.1 \times 10^{11} \text{ N/m}^2$
Material density	7850 kg/m^3	Poisson's ratio	0.3
Damping ratio of each order	1%	Length of main beam section L_0	0.5m
Drag coefficient C_D	1.2	Slope of lift coefficient C'_L	3.5
Lift coefficient C_L	-0.1	Average wind speed V_0	10m/s
Air density ρ	1.225 kg/m^3		

The fluctuating wind speed is synthesized by the harmonic synthesis method through MATLAB language programming, and the Panofsky spectrum is selected as the vertical wind spectrum (model y direction). The longitudinal (z-direction of the model) average wind speed is taken as 5m/s, the surface category is set as D according to the wind resistance specification, and the height from the ground is 35m. The corresponding vertical fluctuating wind turbulence intensity is 0.13, and the lateral (model X direction) attenuation coefficient is 8. Set the sampling frequency to 200Hz (the sampling step is 0.005s) and the duration to 200s. The fluctuating wind load only loads the Scanlan buffeting force without considering the vortex-induced force and self-excited force, and does not load the initial displacement and velocity. The Newmark- β method is used to solve the dynamic response time history of 9 sampling points of the simulation model. Add 20% white noise to simulate the inevitable noise pollution during data acquisition. It should be noted that the random wind response is transformed into the free response by means of the random decrement technique (RDT) for modal identification using the ERA method.

The system order $2n$ setting is incremented from 4 to 98, and the stability diagram is plotted as shown below.

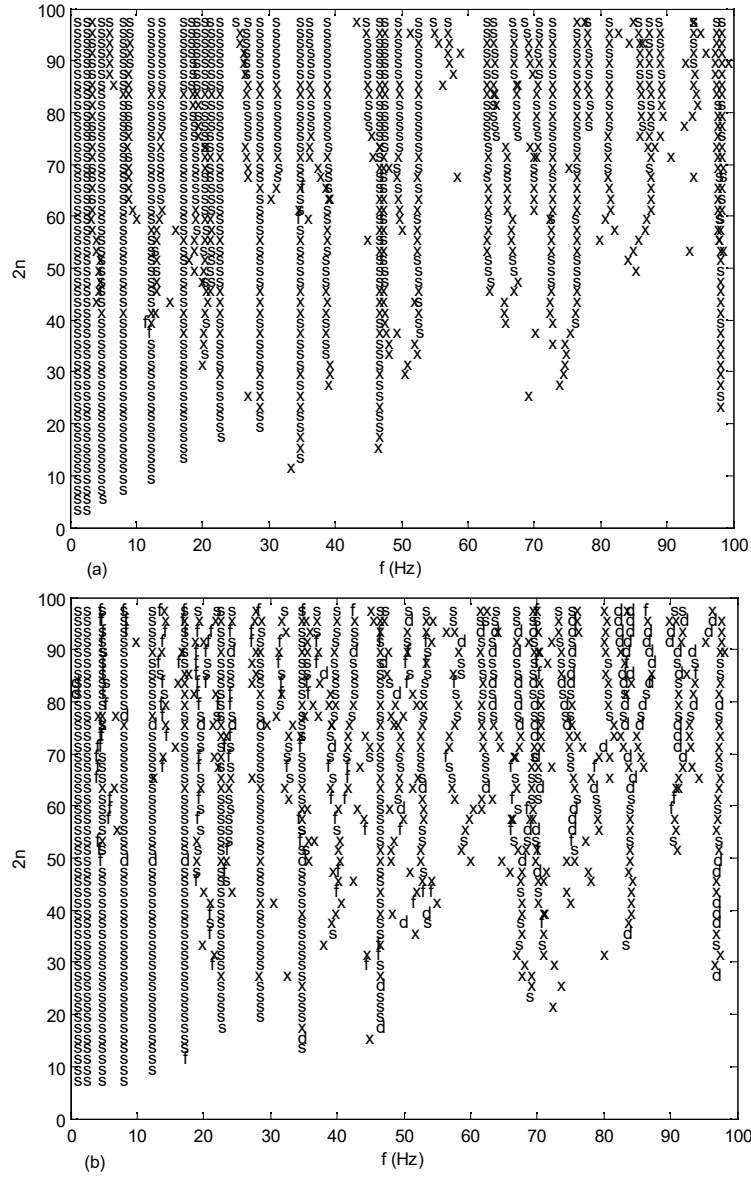
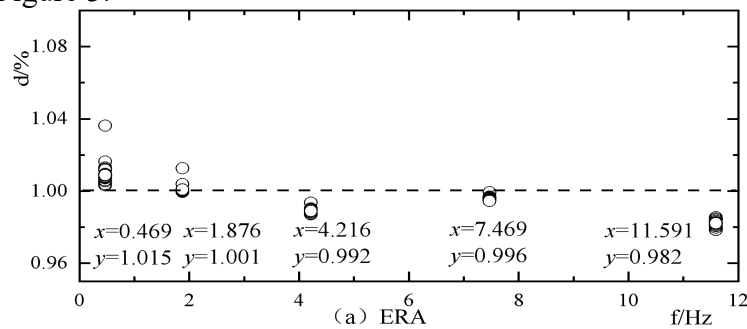


Fig. 2 Stability diagram (a) ERA (b) SSI

In the figure, the character s stands for frequency stability + damping stability, the character f stands for frequency stability, and the character d stands for damping stability. The character x represents frequency instability + damping instability. The stability diagrams obtained by the two methods are basically the same, and the low-order modal stability is very high. In order to improve the stability of high-order modes, it is necessary to further increase the number of measurement points, the sampling frequency and the sampling time.

The s-modes with stable frequency and damping in Figure 2 are clustered, and the clustering results are shown in Figure 3.



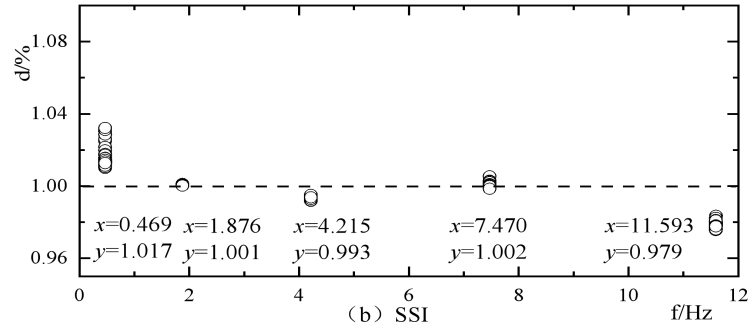


Fig. 3 Diagram of stable clustering modal results (a) ERA (B) SSI

By analyzing the data in the figure, it can be seen that the frequency obtained by the stable clustering method is basically consistent with the theoretical frequency, and the error of the example is within 1%. The dispersion of damping is large, for example, the maximum damping deviation of the first mode can reach 4%, and the damping error after clustering is only 2%. The large deviation of the damping of the fifth mode is due to the fact that four of the nine measuring points are located at the zero amplitude position of the mode shape. The results show that the accuracy of modal identification can be improved by increasing the number of measuring points and arranging the measuring points at the position where the amplitude is the largest.

5. Summary

Compare with that SSI method, the ERA method is more stable because it cannot directly deal with the random response data, It is necessary to provide free response data after conversion with the help of random decrement technology. But to some extent, it also increases the error of modal parameter identification. The SSI method itself provides an outlet for noise and can directly process random response data.

The stable clustering method can effectively eliminate the false unstable modes and further reduce the identification error through clustering. It can effectively reduce the instability of the damping identification results. The example proves that this method is reliable and effective.

In general, without the need for artificial pulse excitation, The stable clustering method can automatically, accurately and efficiently identify the multi-order modal parameters required by wind tunnel tests by using turbulent wind load excitation. Provide guarantee for other subsequent tests.

References

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