

# Prediction formula of blasting vibration under complex terrain based on dimensional analysis

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**Abstract:** The accurate prediction of blasting vibration plays an important role in the evaluation of blasting negative effects and the optimization of blasting design. On the premise of clarifying the influencing factors of blasting vibration propagation, combined with dimensional analysis, based on the traditional Sadovsky formula, through the introduction of horizontal distance and elevation difference factor, the attenuation law of blasting vibration under mountainous terrain conditions is studied, and the blasting vibration prediction formula reflecting mountainous terrain conditions is established. Combined with the measured vibration data of underground powerhouse excavation of Xiamen pumped storage power station in Fujian Province, when the elevation between the measuring point and the explosion source changes greatly, the prediction errors of horizontal and vertical vibration by using the traditional Sadovsky formula are large, about 16 ~ 25% respectively. The prediction errors of vibration prediction formula based on dimensional analysis are small, about 8 ~ 12%. The results show that the particle vibration peak formula based on dimensional analysis can be better applied to the prediction of blasting vibration under complex terrain.

**Keywords:** blasting vibration; dimensional analysis; elevation differences; attenuation; prediction

With the rapid development of the national economy, more and more water conservancy, mining, and transportation projects are being built in China. Drilling and blasting technology is widely used in the excavation of rock masses in the above-mentioned engineering projects due to its high efficiency and economy. However, during the explosion process, the energy released by explosives is not only used for rock fragmentation, but also partially propagates in the form of waves, inducing surface particle vibration and causing damage and destruction to facilities, equipment, and buildings along the propagation path. In order to accurately evaluate the degree of damage and destruction of buildings (structures) under the action of blasting seismic waves and ensure stability and safety, it is extremely necessary to conduct research on the attenuation law of blasting seismic wave propagation.

So far, the research on the attenuation law of blasting seismic wave propagation has received widespread attention from domestic and foreign researchers and engineers. Existing research has shown that there are many factors that affect the propagation attenuation of blasting vibration, such as the maximum single shot explosive quantity, detonation method, and propagation medium properties. Bi Mingya et al. [1] conducted an analysis of the attenuation law of blasting vibration propagation based on existing research results and measured vibration data. The research results pointed out that factors such as terrain and geological conditions, charge shape, and free surface conditions can have a significant impact on the prediction of blasting vibration intensity. Bi Weiguo et al. [2] used analysis methods such as least squares and empirical formulas to preliminarily explore the optimization selection of the attenuation formula for blasting vibration velocity. Jiang Lei [3] took the evaluation of the impact of blasting construction on nearby residential buildings in Fuling Quarry as an example, combined with on-site measured blasting vibration data, and focused on analyzing the attenuation law of blasting vibration and exploring the evaluation of the impact of

blasting on nearby residents in this quarry. Zhang Shixiong et al. [4] evaluated the degree of impact on underground buildings under blasting based on a large amount of on-site measured data, and established the relationship between the peak particle vibration velocity and the single charge quantity and blast center distance. Ling Tonghua, Lin Dachao, and others [5-10] analyzed the attenuation law of blasting seismic wave propagation based on methods such as wavelet transform or Fourier transform.

Most engineering experiments have shown that the Sadovsky formula has high accuracy in predicting the particle velocity of blasting vibration on the ground under flat terrain conditions. However, due to the fact that the formula does not consider the influence of the height difference between the measuring point and the blasting center, the accuracy of using this formula to predict blasting vibration is poor when the terrain and topography of the blasting site change significantly [11]. Therefore, exploring the propagation law of blasting seismic waves under complex terrain and landforms has become a top priority in research, and has a crucial guiding role in the safety evaluation of buildings such as residential buildings under the action of blasting seismic waves. Tang Hai et al. [12] found that the propagation of blasting seismic waves often exhibits a certain elevation amplification effect under convex terrain conditions. Jiang Nan et al. [13] obtained a blasting vibration attenuation formula that reflects the factor of elevation change by analyzing a large amount of measured blasting vibration data and combining it with dimensional analysis theory. Chen Ming et al. [14] analyzed in detail the elevation amplification effect of blasting vibration through a combination of numerical analysis and on-site experiments. Li Haibo et al. [15] pointed out through analysis of measured vibration data that when there are significant changes in terrain and landforms, the predicted blasting vibration speed based on the Sadovsky formula has a significant error, with an average error of up to 59%. The results of the previous analysis indicate that with the variation of the height difference of the propagation terrain, the analysis of the propagation law of blasting vibration using traditional empirical formulas for attenuation of blasting vibration often has significant errors. In order to accurately predict the peak vibration velocity of blasting particles in the key focus area and reasonably evaluate the damage impact of blasting on buildings, it is necessary to conduct analysis of the attenuation law of blasting vibration propagation in complex terrain.

Based on this, this article takes measured blasting vibration data as the research object, combines dimensional analysis theory, and proposes an attenuation formula for blasting vibration that reflects elevation changes on the basis of the traditional Sadovsky empirical formula. By comparing the prediction errors of the improved formula with the Sadovsky formula, the reliability of the improved formula is further demonstrated, to provide reliable theoretical support for the prediction and impact evaluation of blasting vibration under high and steep slopes and complex terrain conditions in the future.

## 1. The attenuation formula for blasting vibration reflecting elevation

As is well known, there are many factors that affect the magnitude of particle blasting vibration velocity, especially in mountainous terrain conditions where more factors need to be considered. It is difficult to fully consider all factors to determine the function relationship of particle blasting velocity. Generally, dimensional analysis is used to consider the influence of the main variables, treat some variables as constants, and then derive the function relationship between the dependent variable and the remaining independent variables. From the above analysis, it can be seen that the blasting vibration speed of particles is greatly affected by geological conditions, blasting source parameters, blasting source distance, and terrain and topography under complex conditions. Considering the particularity of open-air step slopes, the mechanism of elevation difference on blasting vibration amplification effect is studied. The main variables involved in the calculation model of particle blasting vibration speed are summarized in Table 1.

According to the  $\pi$  theorem, the peak vibration velocity of a particle satisfies:  $V = \Phi(Q, w, a, f, R, H, \rho, c, t)$  (1)

In equation (1),  $Q$ ,  $R$ ,  $c$  are independent physical quantities, and the number of independent dimensions  $m=3$ . From this, it can be concluded that if  $\Pi$  represents a dimensionless quantity, then:

In equation (1), the total physical quantity  $n=10$  is involved, where  $Q, R, c$  are independent physical quantities with  $m=3$  independent dimensions. From this, it can be concluded that if  $\Pi$  represents a dimensionless quantity, then:

$$\left\{ \begin{array}{l} \Pi_1 = \frac{V}{Q^\lambda R^\gamma c^\lambda} \quad \Pi_2 = \frac{w}{Q^\lambda R^\gamma c^\lambda} \quad \Pi_3 = \frac{\rho}{Q^\lambda R^\gamma c^\lambda} \quad \Pi_4 = \frac{H}{Q^\lambda R^\gamma c^\lambda} \\ \Pi_5 = \frac{a}{Q^\lambda R^\gamma c^\lambda} \quad \Pi_6 = \frac{f}{Q^\lambda R^\gamma c^\lambda} \quad \Pi_7 = \frac{t}{Q^\lambda R^\gamma c^\lambda} \end{array} \right\} \quad (2)$$

In the formula,  $\lambda, \gamma, \lambda$  are undetermined coefficients.

Table 1 Main variables involved in the calculation of blasting vibration speed

variables	dimension
$w$ : Particle vibration displacement	$L$
$V$ : Particle velocity	$LT^{-1}$
$a$ : Particle vibration acceleration	$LT^{-2}$
$f$ : Particle vibration frequency	$T^{-1}$
$Q$ : the charge mass of maximum delay-interval	$M$
$H$ : Elevation difference between explosion source and measuring point	$L$
$R$ : Horizontal distance between explosion source and measuring point	$L$
$\rho$ : Rock mass density	$ML^{-3}$
$c$ : propagation velocity	$LT^{-1}$
$t$ : Detonation time	$T$

According to the homogeneity theorem of dimensional analysis, since  $\Pi$  is a dimensionless quantity, there is

$$\left\{ \begin{array}{l} \Pi_1 = \frac{V}{c} \quad \Pi_2 = \frac{w}{R} \quad \Pi_3 = \frac{\rho}{QR^{-3}} \\ \Pi_4 = \frac{H}{R} \quad \Pi_5 = \frac{a}{R^{-1}c^{-2}} \quad \Pi_6 = \frac{f}{R^{-1}c} \quad \Pi_7 = \frac{t}{Rc^{-1}} \end{array} \right. \quad (3)$$

Simplification can be obtained

$$\frac{V}{c} = \Phi\left(\frac{w}{R}, \frac{\rho}{QR^{-3}}, \frac{H}{R}, \frac{a}{R^{-1}c^{-2}}, \frac{f}{R^{-1}c}, \frac{t}{Rc^{-1}}\right) \quad (4)$$

Since in dimensional analysis, the powers and products of different dimensionless numbers are still dimensionless, a new dimensionless number  $\Pi_8$  can be established, that

$$\frac{V}{c} = \frac{H}{R} \frac{\rho^{1/3}}{Q^{1/3}R^{-1}} \quad (5)$$

When considering the relationship between the peak velocity of blasting vibration and  $\frac{H}{R}$ , equation (5) can be rewritten as:

$$\ln V = \alpha_1 + \beta_1 \ln\left(\frac{\sqrt[3]{Q}}{R}\right) - \beta_1 \ln\left(\frac{H}{R}\right) \quad (6)$$

Simplified the equation (5), the vibration,  $V$ , can be amended as:

$$V = K\left(\frac{\sqrt[3]{Q}}{R}\right)^{\beta_1} \left(\frac{H}{R}\right)^{\beta_2} \quad (7)$$

## 2. Engineering Examples

The Xiamen Pumped Storage Power Station is located in Tingxi Town, Tong'an District, Xiamen City, Fujian Province, with road mileage of 276km, 86km, and 50km respectively from Fuzhou City, Quanzhou City, and Xiamen City. This power station is located near the load center and adjacent to the main power consumption areas of Quanzhou and Xiamen in the province, with a superior geographical location. The power station is approximately 6km away from the Xiamen 500kV substation in a straight line, and the grid connection conditions are very convenient. The power station is a daily regulation pure pumped storage power station with an installed capacity of 1400MW. The engineering development task is to undertake tasks such as peak shaving, valley filling, frequency regulation, phase regulation, and emergency backup for the Fujian power grid. The layout plan of the site engineering is shown in Figure 1.

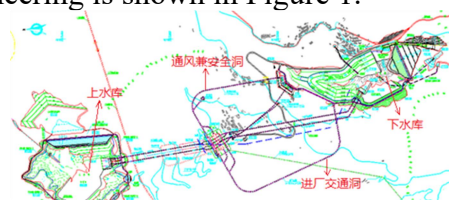
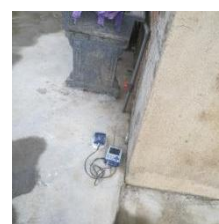


Fig. 1 Planning rendering

Taking advantage of the excavation and construction opportunity of Xiamen Pumped Storage Power Station, blasting vibration monitoring was carried out. Based on the measured blasting vibration data, the attenuation law of blasting vibration propagation under complex terrain was studied. A total of 5 vibration collectors were arranged in this project, and the layout of blasting points is shown in Figure 2.



(a)



(b)

Fig. 2 Layout of blasting vibration monitoring points

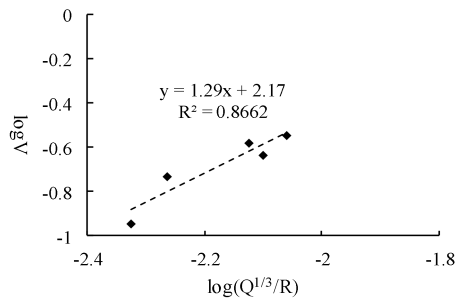
Table 2 Blasting Vibration Monitoring Data

Measurement point number	the charge mass of maximum delay-interval	distance between blasting center and monitoring position /m		Horizontal radial/ (cm/s)	Vertical direction/ (cm/s)	Horizontal tangential direction/ (cm/s)
		Horizontal radial	Elevation			
1#	30	254.5623696	252	0.283	0.368	0.289
2#		296.8619208	255	0.231	0.296	0.201
3#		324.2036397	259	0.264	0.315	0.164
4#		475.1347177	320	0.186	0.237	0.151
5#		556.2481461	352	0.113	0.138	0.078

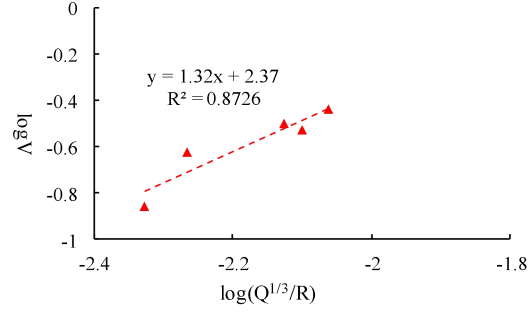
For the study of the attenuation law of blasting seismic wave propagation, the commonly used formula in China is the Soviet Union's Sadovsky formula, which is expressed as follows:

$$Q = K \left( \frac{Q^{1/3}}{R} \right)^\alpha \quad (8)$$

Based on the blasting parameters, the Sadovsky formula (Equation (12)) can be used to fit the measured blasting vibration data. Taking horizontal and vertical directions as examples, the fitting results are shown in Figure 3



(a) Horizontal radial



(b) Vertical direction

Fig.3 Fitting results based on the Sadovsky formula

Regression analysis was conducted on the measured blasting vibration data of the Zhoushan Green Petrochemical Base blasting test in Table 2 using equation (11). The attenuation laws of vibration propagation in the horizontal radial, vertical, and horizontal tangential directions were obtained as follows:

(1) Horizontal radial:

$$V_x = 147.91 * 39.98 * \left(\frac{Q^{1/3}}{R}\right)^{2.23} * \left(\frac{H}{R}\right)^{-1.79} \quad (R^2 = 0.94) \quad (9)$$

(2) Vertical direction:

$$V_z = 234.42 * 1.41 * \left(\frac{Q^{1/3}}{R}\right)^{1.54} * \left(\frac{H}{R}\right)^{-0.81} \quad (R^2 = 0.90) \quad (10)$$

By comparing Figure 3 and equations (13) to (14), it can be found that when fitting the measured blasting vibration data under complex terrain based on the improved vibration attenuation formula (equation (11)), the fitting correlation is significantly improved, which indirectly reflects the important influence of elevation on the attenuation of blasting vibration transmission.

To verify the accuracy of the particle peak vibration velocity attenuation formula considering the influence of elevation proposed in this article, equation (15) was used to calculate and analyze the fitting average error.

$$\varsigma = \sqrt{\frac{\sum_{i=1}^{N_0} (y_i - y_{0i})^2}{N}} \quad (11)$$

In the formula,  $y_{0i}$  represents the vibration results of the measurement point  $i$  test;  $y_i$  is the fitting calculation result.

Compared with the measured values, the difference range of using the Sadovsky attenuation formula to predict the peak vibration of particles is between 16% and 25%; The prediction error of blasting vibration using the attenuation formula considering the influence of elevation (Equation (11)) is 7%~10%, and the prediction accuracy has been significantly improved; The above analysis can demonstrate that using the attenuation formula proposed in this article to fit the peak particle vibration considering the influence of elevation to predict the particle vibration velocity on the blasting surface has better correlation and can more accurately and quickly reflect the basic law of seismic wave attenuation with propagation distance in complex terrain. It can be better used to analyze the attenuation law and prediction of blasting vibration in the field of blasting engineering.

### 3. Conclusion

This article combines the analysis of the measured vibration data system of Xiamen Pumped Storage Power Station in Fujian Province to investigate the influence of complex terrain on the propagation attenuation law of blasting seismic waves. The following conclusions are drawn:

(1) Based on the theory of dimensional analysis, a detailed empirical formula for the attenuation

$$V = K \left( \frac{\sqrt[3]{Q}}{R} \right)^{\beta_1} \left( \frac{H}{R} \right)^{\beta_2}$$

of blasting vibration velocity reflecting elevation changes has been derived,

(2) According to the fitting results of the measured vibration data, it can be concluded that compared with the traditional Sadovsky empirical formula, the proposed blasting vibration propagation attenuation formula considering elevation changes has better fitting correlation and higher accuracy in use.

This article only explores the propagation attenuation law of blasting seismic waves from the perspective of elevation changes in complex terrain. However, the propagation of blasting seismic waves is also affected by structural planes such as internal cracks and fractured zones in the rock mass, and further research should be conducted in this area.

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Declaration of Competing Interest: None

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